Math 1C Quiz 1 Version 1

Fri Oct 7, 2016

NAME YOU ASKED TO BE CALLED IN CLASS: ____________

SCORE: 15 / 30 POINTS   12+3

1. **NO CALCULATORS OR NOTES ALLOWED**
2. **UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS**
3. **SHOW PROPER CALCULUS LEVEL WORK / PROOFS TO JUSTIFY YOUR ANSWERS**

Determine if each of the following converges or diverges. **SCORE:** 3 / 12 PTS

If it converges, determine what it converges to.

If it diverges, write "DIVERGES".

Justify each answer using proper mathematical reasoning, algebra and/or calculus.

[a] $\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}}$

$a_1 = 1$

$a_2 = 3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$

$a_2 = a_1 \cdot r \Leftrightarrow r = \frac{1}{\sqrt{3}} = 3^{-\frac{1}{2}}$

$r = \frac{1}{\sqrt{3}}$ $|r| < 1 \Rightarrow$ converges

$$\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}} = \frac{1}{1 - \frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}}{\sqrt{3}-1}}$$

[b] $\left\{ \frac{\sin n}{e^n} \right\}$ Diverges

$$\lim_{n\to\infty} \frac{\sin n}{e^n} = \lim_{n\to\infty} \frac{\sin n}{n} \cdot \frac{n}{e^n}$$

[c] $\left\{ \frac{n}{\sqrt{1+4n^2}} \right\}$

$$\underbrace{\lim_{n\to\infty} \frac{n}{\sqrt{1+4n^2}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{1}{n^2}+4}}}_{2} = \underbrace{\boxed{\frac{1}{2}}}_{1}$$

$\Rightarrow \left\{ \frac{n}{\sqrt{1+4n^2}} \right\}$ converges

[d] $\sum_{n=1}^{\infty} \frac{2+2^{2n}}{5^n}$ (converges) $= \sum_{n=1}^{\infty} \frac{2+4^n}{5^n}$

$$= \sum_{n=1}^{\infty} \frac{2}{5^n} + \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$$

$$= 2 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n + 4 \sum \left(\frac{1}{5}\right)^n :$$

$r = \frac{1}{5}$, $|r| < 1$ → CONVERGES

$r = \frac{1}{5}$, $|r| < 1$ → CONVERGES

$$= 2 \cdot \frac{1}{1-\frac{1}{5}} + 4 \cdot \frac{1}{1-\frac{1}{5}} = \frac{6 \cdot 5}{4} = \boxed{\frac{15}{2}}$$

Consider the following statements. SCORE: 3 /3 PTS

(i) If $\{a_n\}$ has limit 0, then $\sum_{n=1}^{\infty} a_n$ is convergent

(ii) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\{a_n\}$ diverges

(iii) If $\{a_n\}$ is bounded, then $\{a_n\}$ converges

Which of the statements above are true ? **Circle the correct answer below.**

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[a]	none are true	[b]	only (i) is true	[c]	only (ii) is true	[d]	only (iii) is true
[e]	only (i) and (ii) are true	[f]	only (i) and (iii) are true	[g]	only (ii) and (iii) are true	[h]	all are true

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Find all values of x for which $S = \sum_{n=0}^{\infty} 2^{n+1}(3-x)^n$ is convergent. **You do NOT need to find the sum.** SCORE: 2 /4 PTS

$a_1 = 2(3-x)^0 = 2$

$a_2 = 2^2(3-x)^1 = 4(3-x)$

$a_2 = a_1 \cdot r \iff 4(3-x) = r \cdot 2$

$\iff r = 2(3-x)$

2

$S = \sum_{n=0}^{\infty} 2^{n+1}(3-x)^n$ is convergent if $|r| < 1 \iff |2(3-x)| < 1$

$\iff |3-x| < \frac{1}{2} \iff -\frac{1}{2} < 3-x < \frac{1}{2} \iff \frac{1}{2} > x-3 > -\frac{1}{2}$

$\iff \boxed{\frac{7}{2} > x > \frac{5}{2}}$

Determine if each sequence below is increasing, decreasing or neither. SCORE: 4 /5 PTS

Justify each answer using proper mathematical reasoning and/or algebra.

Your solutions must NOT use derivatives.

[a] $\left\{\frac{5n-11}{2n-5}\right\}$ Decreasing

$\frac{5(n+1)-11}{2(n+1)-5} - \frac{5n-11}{2n-5} = \frac{5n-6}{2n-3} - \frac{5n-11}{2n-5}$

$= \frac{(5n-6)(2n-5)-(5n-11)(2n-3)}{(2n-3)(2n-5)}$

$= \frac{10n^2-25n-12n+30-10n^2+15n+22n-33}{(2n-3)(2n-5)}$

$= \frac{-3}{(2n-3)(2n-5)} < 0$

[b] $\left\{\frac{3n-5}{4n-3}\right\}$ increasing 1

$\frac{3(n+1)-5}{4(n+1)-3} - \frac{3n-5}{4n-3}$

$= \frac{3n-2}{4n+1} - \frac{3n-5}{4n-3}$ 1

$= \frac{(3n-2)(4n-3)-(3n-5)(4n+1)}{(4n+1)(4n-3)}$

$= \frac{12n^2-17n+6-12n^2+17n+5}{(4n+1)(4n-3)}$

1 $= \frac{11}{\underbrace{(4n+1)}_{>0}\underbrace{(4n-3)}_{>0}} > 0$ 1

$\boxed{n \geq 1}$